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An anisotropic damage model of foams on the basis of a micromechanical description

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The mechanical behavior of open-cell foams may be modeled either on a microscopic or a macroscopic scale. In the first case, the behavior of the individual cell walls is described by beam models, while in the second case a continuum mechanical approach is applied. Both approaches have different advantages: On the one hand, the microscopic approach allows for a simple formulation of the constitutive equations but requires detailed knowledge of the heterogeneous microstructure, e.g. geometrical data of the beams and of the topology, and becomes numerically expensive for large structures. On the other hand, the macroscopic approach leads to efficient computations but requires more complicated constitutive equations, if e.g. anisotropy is taken into account. In the present contribution the advantages of the microscopic and macroscopic descriptions are combined by a numerical so-called second order homogenization scheme. Therefore, a small but representative element of the microstructure consisting of a few beam elements is chosen and attached to the quadrature points of the macroscopic finite element model. The macroscopic model is formulated in the framework of a Cosserat continuum, which allows to take care of size effects. The macroscopic strain and curvature tensors are projected onto the microstructure leading to a deformation mode of the beam ensemble. The resulting forces and moments in the beams are homogenized by an appropriate averaging procedure defining the corresponding stresses and couple stresses on the macroscale. In this approach, anisotropy is included in a natural way choosing an anisotropic distribution of the beams in the testing volume element (TVE). In addition, damage is described on the microscopic level of the individual beams.

1. Introduction

The mechanical behavior of open-cell foams may be modeled either on a microscopic or on a macroscopic scale. In the first case, the behavior of the individual cell walls is described using beam models [1, 2] while in the second case a macroscopic continuum theory is applied. As discussed in the literature [3–5], the equivalence of both approaches requires an extended continuum mechanical setting on the macroscopic scale to describe the size effects as well as the boundary layers observed in cellular materials. As motivated by the microscopic investigation the rotational degrees of freedom (rotation of the cross-section of the beam elements) and the related moments have to be taken into account. Therefore, a Cosserat-type or micropolar continuum [6, 7] is an appropriate choice on the macroscale, see also [8] and the literature cited therein. Furthermore, this approach is able to resolve the inconsistencies between effective elastic parameters determined either in tension, in bending or in shear [1], i.e. it solves the problem refered to in data sheets given by foam manufactures who list different values for the mechanical moduli depending on the loading conditions [9].

The approaches on the different scales are linked to each other by a second order homogenization procedure, which relates microstructural properties to extended continuum theories [10, 11]. In classical homogenization procedures a Representative Volume Element (RVE) is chosen which reflects the statistical properties of the microstructure [12, 13]. The locally fluctuating properties of the microstructure are replaced by effective quantities on the macroscale. This leads to effective moduli which are obtained by appropriate averaging procedures. The determination of effective moduli is usually restricted to linear problems because analytical solutions are required on the scale of the RVE. A detailed discussion concerning the homogenization approach may be found in the literature, e.g. in [12–14]. Note that in the present study, a Testing Volume Element (TVE) according to Huet [15] is chosen. As

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a main difference, the TVE is not representative for the specimen as a whole but reflects local properties. Therefore, the TVE is in general smaller than a RVE and is able to resolve boundary layers.

In the present work, a numerical homogenization strategy is proposed which combines the advantages of the purely microscopic and the purely macroscopic approaches. On the one hand, the microscopic approach allows for simple constitutive equations taking into account anisotropy in a natural way, on the other hand, the macroscopic approach is numerically efficient even for large engineering problems. Therefore, a so-called FE^2 approach is applied [10, 11, 16] in the context of a second order homogenization scheme. This higher order homogenization links the microstructural properties to an extended continuum theory [10, 11] while a first order homogenization leads to a classical continuum formulation on the macroscale. As the main idea of the FE^2 procedure an explicit microstructure is attached to each quadrature point of the finite element discretization of the macroscopic model. From the finite element analysis the macroscopic strain, and in the present case, the macroscopic curvature are known at each quadrature point of the finite element model. These kinematic quantities are projected onto the boundaries of the attached microstructure consisting of certain beam elements. For this TVE the forces and moments in the beams are computed according to the Dirichlet data obtained from the projection of the macroscopic kinematic quantities onto the boundaries of the TVE. Finally, applying the second order homogenization procedure, the macroscopic stresses and couple stresses are derived form the microscopic distribution of the forces and the moments. The idea is sketched in Fig. 1. In the present approach anisotropy is naturally taken into account by an anisotropic distribution of the beams in the TVE and damage is included in the model by a microscopic formulation on the beam level.

The paper is organized as follows: In Section 2, the microscopic model is briefly discussed. This includes the choice of the beam topology in the TVE and the formulation of the microscopic constitutive equations. In Section 3, the homogenization procedure is discussed. This procedure transfers the forces and the moments from the microscale to the macroscale resulting in stresses and couple stresses. Finally, in Section 4, some examples are discussed. The paper is closed with a summary and some conclusions.

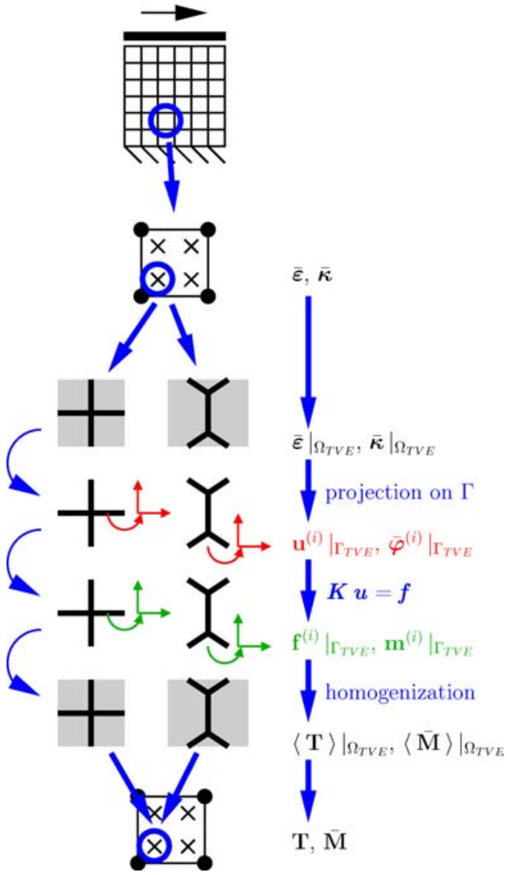

Figure 1 Schematic sketch of the applied FE^2 approach.

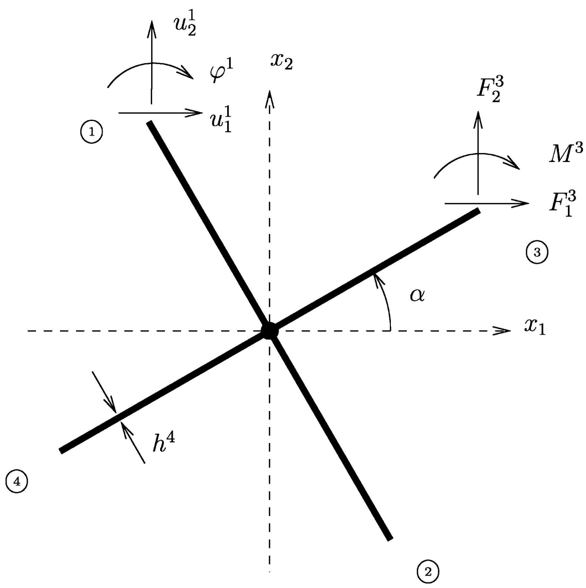

Figure 2 TVE consisting of four beam elements.

2. Microscopic model

Open-cell foams can be modeled by beam elements on the microscopic scale. In the present contribution, each cell wall is replaced by a Timoshenko beam element allowing for independent displacements and rotations. Once, if the microscopic structure is known, e.g. from computer tomography, the global behavior of the foam is computed as a boundary value problem following the concepts of structural mechanics, e.g. [17, 18].

In the present study, a two dimensional lattice model is chosen on the microscale for simplicity. Furthermore, the theoretical description is restricted to a geometrically linear setting with small displacements and rotations. The TVE consists of four identical Timoshenko beams which are rigidly joined in the center point, cf. Fig. 2. Each of the beams has the length l, the cross-section $A^i = b\,h^i$ and a bending stiffness EI. The orientation of the TVE may be tilted against the vertical by an angle α.

According to the results found in [3] the TVE has to be small in order to resolve boundary layers. Therefore, the smallest possible unit cell is chosen in our model.

The mechanical behavior of the TVE is completely described by the displacements u_1^i, u_2^i and by the rotation φ^i of the beam end points i. The resulting forces F_1^i, F_2^i and the resulting moment M^i can be computed from the geometrical properties following the concepts of elementary structural mechanics, cf. [1]. It can be found that the structural behavior of the TVE is anisotropic, therefore, if all TVE are oriented in the same way, macroscopic anisotropy is automatically included in the approach.

Finally, damage is included into the model which is taken into account as a reduction of the effective cross-section of the beam elements. More general, the formulation is based on the introduction of scalar-valued damage variables d^i for each of the four beams in the TVE, which are determined by an evolution equation [19]. In the present study a rate independent ansatz of the form

$$d^i = d_\infty [1 - \exp(-f^i/\eta)] \tag{1}$$

is chosen, which results from the integration of an appropriate evolution equation. Therein, d_∞ is the maximum value of damage that may occur, η is an additional material parameter and

$$f^i = \begin{cases} 0 & \text{if } |\sigma^i| < \sigma_{\text{crit}}, \\ |\sigma^i - \sigma_{\text{crit}}| & \text{if } |\sigma^i| \geq \sigma_{\text{crit}}. \end{cases} \tag{2}$$

According to (2) a simple normal force criterion is chosen which drives the evolution of the damage variable. Therefore, σ^i is the average normal stress in the beam element i related to the normal force N^i. And σ_{crit} is a threshold which has to be reached before the evolution of damage starts. Following the classical interpretation of d^i allows for an update of the beam's cross-section according to

$$A^i = (1 - d^i)\, A_0^i \rightarrow h^i = (1 - d^i)\, h_0^i, \tag{3}$$

where A_0^i and h_0^i are the cross-section of the undamaged beam element i and the height, respectively. In the two-dimensional problem under study it is assumed that the width b^i of the beam element does not change. Note that more general assumptions concerning the damage model may be taken into account very easily.

3. Numerical homogenization

The idea of a homogenization procedure is to replace highly inhomogeneous fields on a small scale by smooth effective fields on a larger scale. The concept is based on scale separation arguments also known as the so-called MMM principle [14], cf. Fig. 3.

On the microscale δ each beam is described as a continuum. In a first step these continua are replaced by structures of one-dimensional beam elements on the mesoscale Δ. In the present context, the TVE belongs to this mesoscale. As a consequence, the inhomogeneous distribution of stresses in the beams is replaced by forces and moments according to the Timoshenko beam theory. In a second step the TVE is interpreted as a material point on the macroscale D, i.e. the smallest entity representing the physical properties of the material on the respective scale. Therefore, the discontinuous distributed forces and moments on the mesoscale are finally replaced by a continuous distribution of stresses and couple stresses on the macroscale. Note in passing, that a couple stress has to be present on the macroscale if the bending moments in the beams are of significance [8], see also the discussion in [20–22] in the context of granular media.

Starting point of the derivation of the required homogenization procedure is the definition of the average stress and couple stress

$$\langle \mathbf{T} \rangle = \frac{1}{V} \int_V \mathbf{T}\, \mathrm{d}v, \quad \langle \bar{\mathbf{M}} \rangle = \frac{1}{V} \int_V \bar{\mathbf{M}}\, \mathrm{d}v. \tag{4}$$

Following the usual argumentation [4] the volume integrals are transfered to surface integrals. Finally, in the case of discrete microstructures, the evaluation of the surface integrals leads to an summation over the beam end points

$$\langle \mathbf{T} \rangle = \frac{1}{V} \sum_i \mathbf{f}^i \otimes \mathbf{l}^i, \quad \langle \bar{\mathbf{M}} \rangle = \frac{1}{V} \sum_i \mathbf{m}^i \otimes \mathbf{l}^i. \tag{5}$$

In these expressions $\mathbf{l}^i$ are the branch vectors from the center of the TVE towards the beam ends on the boundary. Furthermore, $\mathbf{f}^i$ and $\mathbf{m}^i$ are the forces and the moments, respectively, acting on the beams at the boundary of the TVE. Taking into account the moments in the

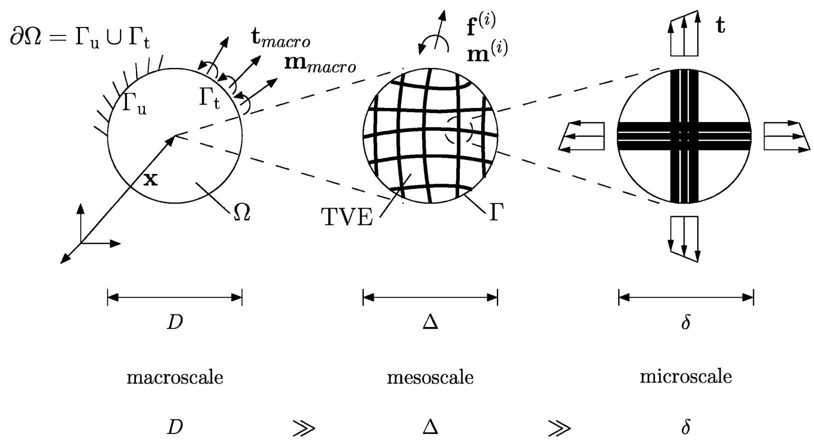

Figure 3 Macro-, meso- and microscale according to the MMM principle.

beam elements requires to enhance the continuum mechanical formulation on the macroscale. Therefore, following the presented homogenization approach leads to a Cosserat or micropolar theory as the equivalent description. The same ideas may also be developed for homogenization strategies in particle assemblies and lead to the same structures [22, 23].

In addition to the homogenized stress and couple stress homogenized kinematic quantities like strain and curvature can be defined [4] but the computation of these quantities is not necessary in the framework of the proposed FE^2 procedure. The macroscopic strain and curvature are the input quantities for the displacement driven homogenization procedure while the stress and couple stress obtained from Equation 5 are the output, cf. Fig. 1.

4. Macroscopic model

As motivated by the homogenization procedure (5), a micropolar or Cosserat model has to be formulated. While in a standard or Boltzmann continuum the material point is a mathematical point, it is assumed to be a microscopic rigid body in the Cosserat formulation [8, 24]. As a consequence, material points are able to undergo not only displacements $\mathbf{u}$ but also rotations $\bar{\boldsymbol{\varphi}}$. Therefore, they transfer stresses $\mathbf{T}$ and couple stresses $\bar{\mathbf{M}}$ which are conjugated to strains $\bar{\boldsymbol{\varepsilon}}$ and curvatures $\bar{\boldsymbol{\kappa}}$, respectively. A detailed discussion of this approach may be found in the text books by Eringen [8] and Nowacki [7] and references therein.

For simplicity, the present model is restricted to small deformations, i.e. to small displacements and small rotations. This assumption was already taken for the microscopic model. In this case, the strain and the curvature are derived in the following way from the displacement field $\mathbf{u}$ and the rotational field $\bar{\boldsymbol{\varphi}}$:

$$\bar{\boldsymbol{\varepsilon}} = \operatorname{grad}\mathbf{u} + \overset{3}{\mathbf{E}}\cdot\bar{\boldsymbol{\varphi}} = \bar{\boldsymbol{\varepsilon}}_{\text{sym}} + \bar{\boldsymbol{\varepsilon}}_{\text{skw}} \quad \text{and}$$
$$\bar{\boldsymbol{\kappa}} = \operatorname{grad}\bar{\boldsymbol{\varphi}}. \tag{6}$$

The gradient operator grad ($\bullet$) takes the derivative with respect to the position $\mathbf{x}$ of a material point. $\overset{3}{\mathbf{E}}$ is the Ricci tensor (permutation tensor). The symmetric part of the Cosserat strain

$$\bar{\boldsymbol{\varepsilon}}_{\text{sym}} = \frac{1}{2}(\operatorname{grad}\mathbf{u} + \operatorname{grad}^T\mathbf{u}) \tag{7}$$

is identical with the well-known strain tensor of linear elasticity while the skew-symmetric part

$$\bar{\boldsymbol{\varepsilon}}_{\text{skw}} = \frac{1}{2}(\operatorname{grad}\mathbf{u} - \operatorname{grad}^T\mathbf{u}) + \overset{3}{\mathbf{E}}\cdot\bar{\boldsymbol{\varphi}} \tag{8}$$

represents the difference between the continuum rotation related to the displacement field and the independent field of rotations $\bar{\boldsymbol{\varphi}}$ [25].

The related equilibrium conditions are derived from the balance of momentum in its quasi-static form

$$\mathbf{0} = \operatorname{div}\mathbf{T} \tag{9}$$

and from the balance of moment of momentum in the form

$$\mathbf{0} = \operatorname{div}\bar{\mathbf{M}} + \mathbf{I}\times\mathbf{T}, \tag{10}$$

neglecting body forces and body couples. Therein, div ($\bullet$) is the divergence related to grad ($\bullet$) and $\mathbf{I}\times\mathbf{T}$ represents twice the axial vector of the tensor $\mathbf{T}$. Note, that in general the stress tensor is not a symmetric tensor in the framework of a micropolar theory.

In general, constitutive equations have to be formulated relating stresses and couple stresses to strains and curvatures. In the present second order FE^2 approach the phenomenological constitutive equations are replaced by the solution of an appropriate boundary value problem on the scale of the TVE and the homogenization scheme, cf. Fig. 1. Details are discussed in [26].

During the computation the stress and the couple stresses have to be evaluated from a microscopic boundary value problem at each quadrature point for given values of macroscopic strains and curvatures. Replacing the constitutive equations by the homogenizations approach leads to the following strategy, which is already shown in Fig. 1 for quadrilateral and hexagonal microstructures: A TVE is attached to each integration point of the finite element mesh. It is assumed that the strains and curvatures as obtained from the macroscopic computation are constant in the domain of the microstructure, therefore, they are related to a linear distribution of the microscopic kinematic variables, i.e. the displacement and the rotation of the Timoshenko beam elements. The projection of these linear distributions onto the boundary of the TVE leads to a formulation of the microscopic boundary value problem which can be solved for a given microstructure. From this boundary value problem the forces and moments in the beams are computed and homogenized by the proposed method. The equivalent stresses $\langle\mathbf{T}\rangle$ and the equivalent couple stresses $\langle\bar{\mathbf{M}}\rangle$ are identified with the macroscopic stresses $\mathbf{T}$ and couple stresses $\bar{\mathbf{M}}$, respectively, and transfered from the level of the TVE to the macroscopic finite element computation. This procedure is known as FE^2 approach [10, 11, 16].

5. Example

As a first example, a tension test of a quadratic specimen with circular hole is investigated. The computation is performed for a quarter of the plate as shown in Fig. 4. The boundary conditions at the left hand side and at the bottom of the specimen reflect the symmetry conditions while the displacement controlled load is applied on the top of the specimen. The right hand side is unloaded and moves unconstraint. The attached microstructure consists of four beams rigidly connected at the center node. Two of them are oriented horizontally

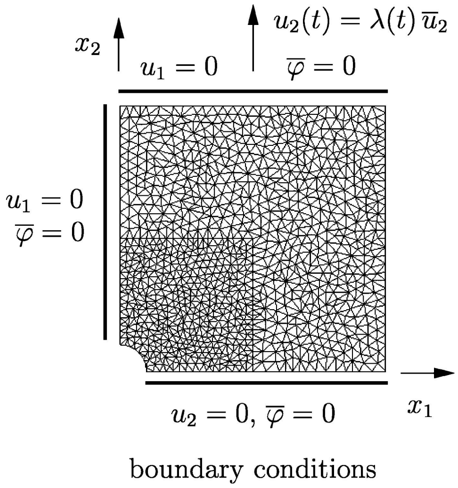

Figure 4 Plate with a hole, definition of boundary conditions.

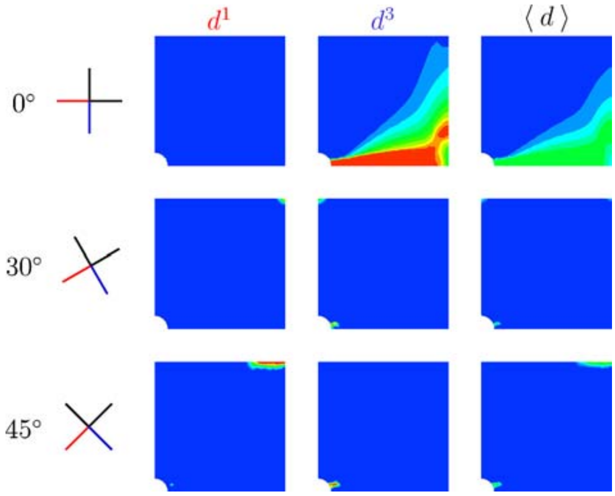

Figure 6 Evolution of the damage variables depending on the microstructural orientation.

while the other two are oriented vertically, respectively. The microstructure corresponds to Fig. 2 setting $\alpha = 0$.

The development of the stress distribution around the hole for increasing loads is shown in Fig. 5. In the first column the development of the horizontal stress T_{11} is shown while in the second column the vertical stresses T_{22} are shown. It can be seen that the hole leads to a stress concentration. Furthermore, the symmetric part of the shear stress $T_{\text{sym}} = \frac{1}{2}(T_{12} + T_{21})$ is shown in the third column. Finally, in the fourth column, the damage variable d^3 which is related to one of the vertical beam elements is shown. According to the normal force criterion equations (1), (2) in the microscopic damage model, the maximum values arise at the right hand side of the hole while the hole itself leads to stress shielding on the left hand side of the boundary, i.e. the vertical stress component is zero in the area located on top of the hole.

In a second example the influence of the orientation of the microstructure is studied in a displacement driven boundary value problem as indicated in Fig. 4. Therefore, three cases are investigated as indicated in Fig. 6. Note that rotating the microstructure decreases its stiffness. Thus, the maximum stresses and maximum damage arise under a 0° orientation of the structure. The upper row shows the damage in a horizontal beam element 1, in a vertical beam element 3 and the average value. In the second row the microstructure is rotated 30° counterclockwise and in the last row 45°. It can be seen that the evolution of the damage strongly depends on the orientation of the microstructure. In the first case damage is mainly observed in the vertically oriented beam elements. Furthermore, it is located in a region close to the bottom of the specimen. The evolution of the damage starts directly at the hole and a mode 1 crack would open if $d^3 = 1$ is reached. Due to limiting the maximum damage to $d_{\max} = 0.9$ the thickness of the damaged zone increases by distributing damage over neighboring elements. For the 45° rotated structure damage starts in the upper right corner. In this case the maximum damage $d_{\max}$ is not reached leading to a narrow damaged zone. Finally, for the structure at 30° the damage evolution is minimal.

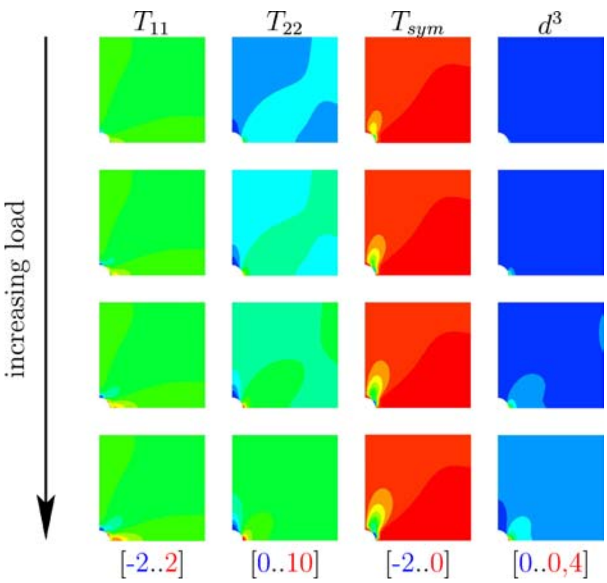

Figure 5 Stress distribution around the hole.

In a final example the damage evolution for stochastically oriented microstructures is investigated. Three ensembles of stochastically oriented microstructures are generated as shown in the left column of Fig. 7. It can be observed that the damage evolution takes place in the region close to the hole and close to the upper right corner according to the damage localization either related to the vertically aligned beam elements or to the elements under 45° orientation. The distribution of the averaged damage variable $\langle d \rangle$ is nearly the same in all three cases, cf. Fig. 7. As expected, the global behavior becomes isotropic while in the previous example a strong dependency on the angle of alignment of the microstructures was found.

On the one hand, as pointed out in [3] the microscopic beam model and the macroscopic Cosserat model show similar results in the range of elastic deformation, on the other hand it is shown in [27] that the same is true comparing FE^2 and the Cosserat formulation. Therefore, it can be concluded that beam models, their FE^2 implementation and Cosserat models describe the same physical effects.

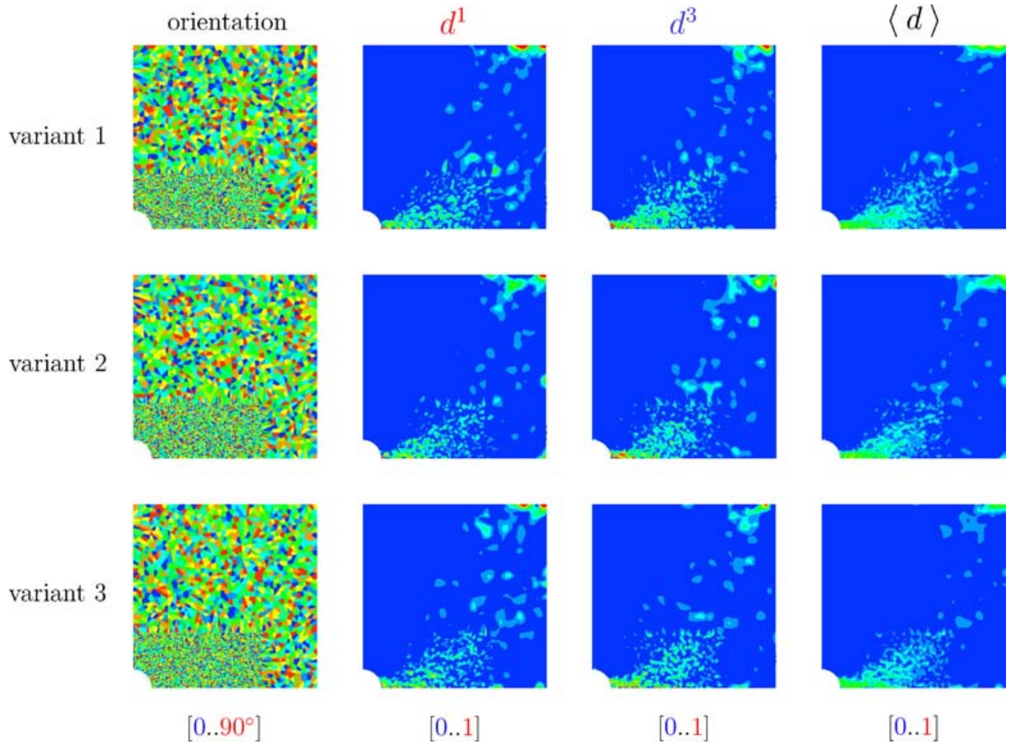

Figure 7 Stochastic distribution of the microorientation and evolution of damage.

6. Conclusions

In the present paper a FE^2 approach is proposed to model the behavior of a cellular solid on the macroscale. While the TVE on the mesoscale consists of an ensemble of Timoshenko beams a second order homogenization scheme is applied to transform the discrete forces and moments in the beam elements into macroscopic stresses and couple stresses. Based on the introduction of displacements and rotations as independent kinematic degrees of freedom a Cosserat theory is chosen as an appropriate continuum mechanical model. This allows to take boundary layers and size effects into account. Finally, in the framework of FE^2, the macroscopic constitutive equations are replaced by the homogenization procedure, while the constitutive equations are formulated on the microscale of the individual beams. Therefore, macroscopic anisotropy is simply obtained by a choice of an anisotropic distribution of the beam elements in the TVE. Furthermore, in this context, the anisotropic evolution of damage is easily formulated. Some examples demonstrate the applicability of the proposed model.

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